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## **Application Note**

# **AL100 Transient & Frequency Responses**



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July 16, 2015

# Application Note

## AL100 Transient and Frequency Responses

INTRODUCTION: This application note discusses the internal signal processing of the AL100, and its effect on the device's transient response and frequency response.

### AL100 Signal Processing

The AL100 internally samples at a fixed rate of 100Hz (1 sample every 10ms). In this note, the internal sample rate will be called "Raw Sample Rate" to distinguish it from the sample rate that is specified by the user. Likewise, the data obtained at the Raw Sample Rate is called "Raw Samples".

The Raw Sample Rate is not exact. The accelerometer's manufacturer has no specification on the accuracy of the sample rate. A method for measuring the actual sample rate is described in the AL100 Manual. In the following, where scaled frequencies are involved, it should be understood that the scaling is with respect to the actual raw sample frequency, not the typical value used in this note.

"N" Raw Samples are averaged in an averager. The user can specify N within the range of 4 to 250. Each averaged set of N samples is then output as a "Sample" (a kind of "sub-sample" of the raw samples). Thus, the output samples occur at a rate of  $100\text{Hz}/N$ ; this is called the "effective sample rate".

To maintain the highest possible resolution while still dealing with integer values, the averager is designed to behave a little different than a normal average. For values of  $N < 9$ , the raw sum is written to the memory card. Thus, none of the sum is discarded. The user can, if desired, do that division as part of post-experiment processing; most likely, this will be in a computing environment that handles decimal fractions more easily. For values of N greater than 8, the sum (in the averager) is multiplied by  $8/N$  and the fractional part is discarded. This is done in the order that results in no lost information other than the discarded fractional part; that is, the value is multiplied by 8 first (by left-shifting 3 times), then the value is divided by N.

For determining a vibration spectrum, the scaling of the values is usually of minor importance. It does become more important if spectra acquired at different effective sample rates are to be compared, and that comparison has to compare the amplitudes of different frequency components.

The moving average filter is one of a set of equivalent filters that are “optimum” for reduction of noise in data (1). These filters are characterized as “low-pass”. That is, they pass low frequency signals while attenuating higher frequency signals. But, these filters do a relatively poor job of removing higher frequencies compared to other digital filters; their strong point is excellent “smoothing” or the reduction of noise.

### Transient Response

For people who are used to continuous signals characterized by first or second order linear differential equations, the time response of a moving average filter may seem a bit odd. Figure 1 shows the response of an  $N=8$  filter to a unit step input that happens near the middle of an averaging interval.

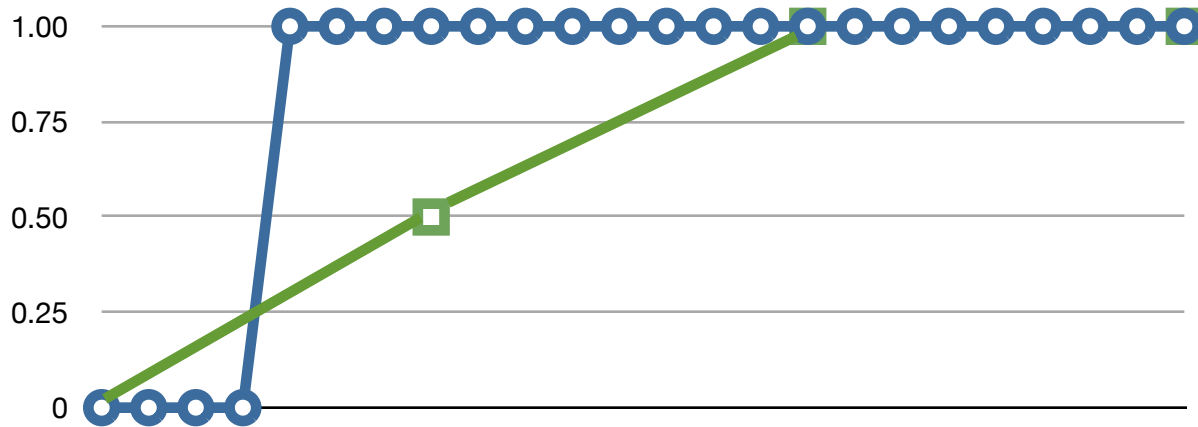


Figure 1 Step Response

Note that the value of output sample that is part way between 0 and 1 will depend WHERE the step occurs within the averaging “box”. If it happens as the very first sample of the average box, then the output will go from 0 to 1 in one output sample (delayed by  $N$ , of course). If the step occurs within an average box, then there will be an intermediate value generated, as shown in the figure. Thus, the step response will always take 1 or 2 output samples to reach 1, no matter what the value of  $N$ , depending on where the step occurs.

There is no “exponential rise time” as there is with continuous signal processing (or other types of digital processing).

## Some System Terminology

Suppose that there is a sinusoidally varying acceleration at some very low frequency. Suppose that this acceleration has an amplitude of "A" (that is, it varies from +A to -A, sinusoidally). We know that, since the sensor (the AL100) is linear, the output will vary sinusoidally and the output will vary from +O to -O.

The relationship between the input and the output might be described in various ways. To some, this is simply a "calibration factor". To others, it is "system gain". In either case, this number is the measure of output units per input unit. For the AL100, this might be "counts per g", that is, the output value at 1g of acceleration.

In the following, the term "frequency response" is used. This term describes how the calibration factor or system gain varies with the frequency of the input signal. It is usually normalized to some frequency. In the case of a "low pass" frequency response, which is what the AL100 has, we will compare the ratio of the system gain at some higher frequency to the system gain at some very low frequency.

Some readers might ask: "Isn't this really artificial, since the motions we are measuring are quite far from sinusoidal?". This may appear to be the case, but it is not. We know, from Fourier Analysis, that periodic non-sinusoidal signals are made up of the sum of many frequencies. Since the sensor is linear, the output will also be the sum of the same frequencies. However, since the system gain is not the same at all frequencies, the relative strength of the different frequencies will be different in the output compared to the input.

The simple fact is that the term "frequency response" only applies to sinusoidal signals. It is a ratio of ratios. It compares the ratio between the output and the input (the system gain, which is a ratio) at different frequencies. And, for a low-pass response, this comparison takes the form of ratio of the system gain at high frequencies to the system gain at low frequencies.

## Frequency Response

The frequency response of a moving average is also somewhat different than for a continuous system. We can, however, use much of the same language.

The magnitude of the frequency response of a moving average filter is given by (2)

$$H[f] = \left| \frac{\sin(\pi f N)}{N \sin(\pi f)} \right| \quad \text{EQN 1}$$

where N is the average size (the sample rate divisor) and f is the signal frequency NORMALIZED TO THE SAMPLE FREQUENCY. That is, if the sample frequency is 100Hz and the signal frequency is 5Hz, then  $f = 5\text{Hz}/100\text{Hz} = .05$ . The maximum allowed useful value of f is 0.5 which is set by the Nyquist Sampling Theorem as applied to the raw sample rate. For  $f = 0$ ,

$H[f] = 1$ . Figure 2 shows the response when  $N = 10$  and  $N = 20$  (the average size is 10 or 20 and the effective output sample rate is  $100\text{Hz}/10 = 10\text{Hz}$  or  $100\text{Hz}/20 = 5\text{Hz}$ ).

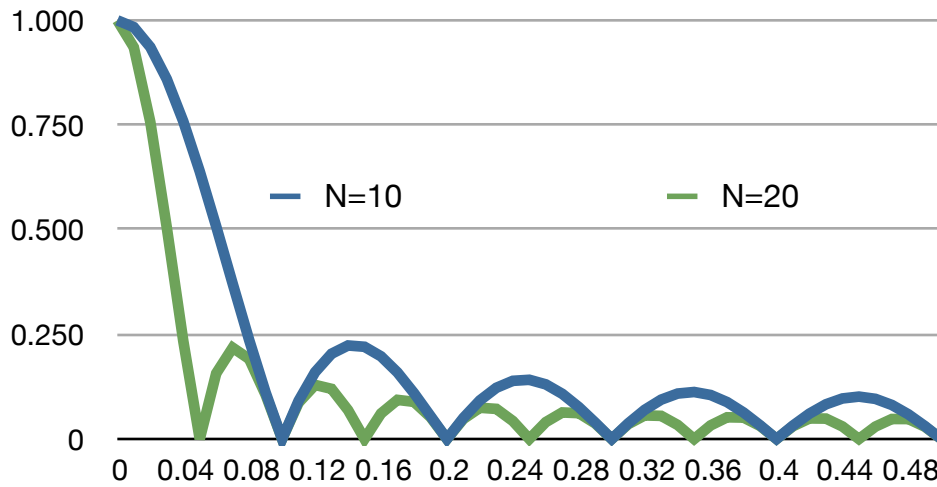


Figure 2 - Frequency Response,  $N=10$  &  $N=20$

Some interesting behavior appears in Figure 2. There are a whole sequence of “nulls” in each frequency response plot. This probably ought to have been expected from EQN 1 since there is a sin function in the numerator of the expression. These nulls appear at frequencies given by

$$\sin(\pi f N) = 0 \quad \text{EQN 2}$$

These occur when

$$\pi f N = K \pi \quad (K = 1, 2, 3, \dots) \quad \text{EQN 3}$$

The lowest of these, or the first null, is when  $K = 1$ , or

$$f = 1/N \quad \text{EQN 4}$$

This is logical, because an average over exactly 1 cycle of a sine will be zero. In fact, these “nulls” occur at exactly the output sample rate because there is one output sample per average “box”.

**Frequency Response at Output Nyquist Frequency:** Next, lets consider what the frequency response is at the Output Nyquist Frequency. The output Nyquist Frequency is  $(1/N)/2$  since  $1/N$  is the output sample rate. Lets call this frequency,  $F_{no}$  (Nyquist Output Frequency). Thus, we want to find (from EQN 1)

$$H[F_{no}] = \left| \frac{\sin(\pi (0.5/N) N)}{N \sin(\pi (0.5/N))} \right| \quad \text{EQN 5}$$

$$H[F_{no}] = \left| \frac{\sin(\pi/2)}{N \sin((\pi/2)/N)} \right|$$

But,  $\sin(\pi/2) = 1$ , and  $\sin((\pi/2)/N)$  is positive for all positive  $N$ , we have

$$H[F_{no}] = 1 / N \sin((\pi/2)/N) \quad \text{EQN 6}$$

The next figure shows this relationship, with the value of  $H(F_{no})$  plotted against  $N$ , the size of the average box and  $N$ , the ratio of the raw sample rate to the output sample rate. For values of  $N$  greater than about 25, the indicated output signal at the output Nyquist Frequency (one-half the Output Sample Rate) is about 0.637. As  $N$  is reduced toward 4 (the minimum for the AL100), the signal rises to about 0.654.

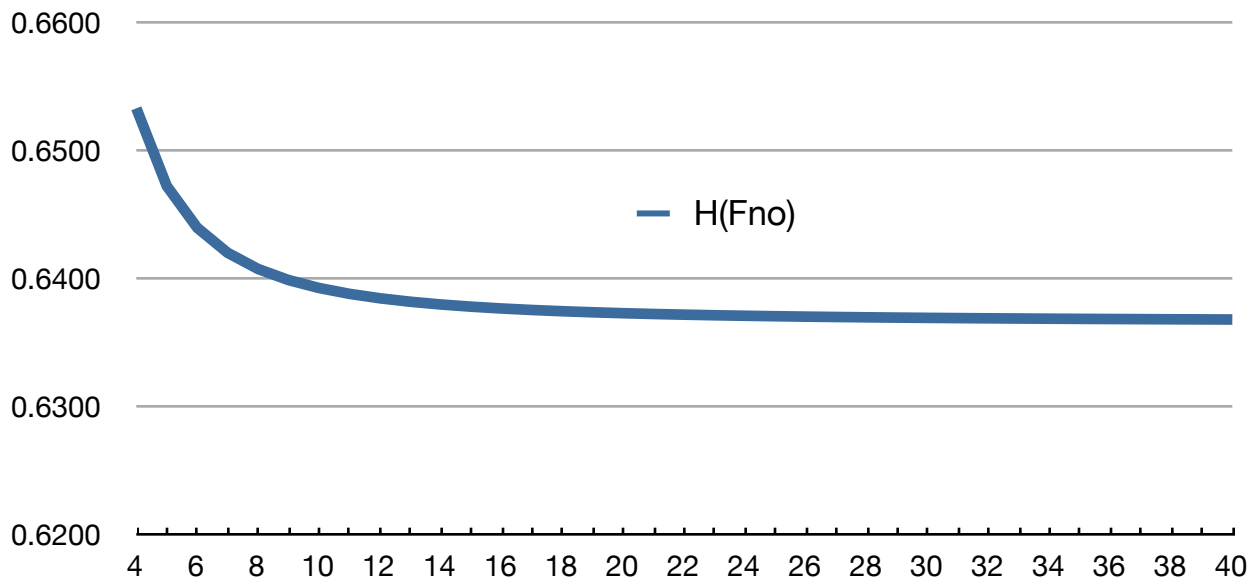


Figure 3 - Frequency Response at the Output Nyquist Frequency vs N

What does this mean? Suppose that we have a sinusoidal acceleration at some very low frequency, and the sensor output shows a numerical amplitude of  $A$ . Now, let's increase the frequency of our acceleration, while holding the physical amplitude of the acceleration constant, up to the Output Nyquist Frequency (which is the highest frequency that we have any hope of properly measuring). If  $N$  is above 25, the indicated numeric output signal will be only 63.7% of what it was at the very low frequency.

**Bandwidth:** Another useful quantity might be "bandwidth". In a low-pass response in a continuous system, this is often taken to be the frequency at which the response falls to  $1/\sqrt{2}$  of the low frequency response; you may know this as the "3db bandwidth". That is, if the low

frequency response is 1, then this is the frequency at which the response has decreased to 0.707.

Note that this is ALWAYS a lower frequency than the Output Nyquist Frequency because the system gain increases steadily at the frequency is decreased below the output sample frequency. Since the system gain is around 0.64 at the Output Nyquist Frequency, the point at where the system gain is 0.707 MUST be at a lower frequency.

EQN 1 is not readily solved, in closed form, to determine the value of "f" that produces this response. However, we can find the frequency value numerically. That result is shown in Figure 4. In this figure, the bandwidth is stated as a fraction of the Nyquist Output frequency, Fno (which is half the output sample frequency). To get this result, the following equation is solved

$$H[f] = | \sin(\pi f N) / N \sin(\pi f) | \quad \text{EQN 1}$$

$$1/\sqrt{2} = | \sin(\pi m/2) / N \sin(\pi m/(2N)) | \quad \text{EQN 7}$$

where m represents the fraction of the Output Nyquist Frequency at which the system gain is  $1/\sqrt{2}$  and N is the average size.

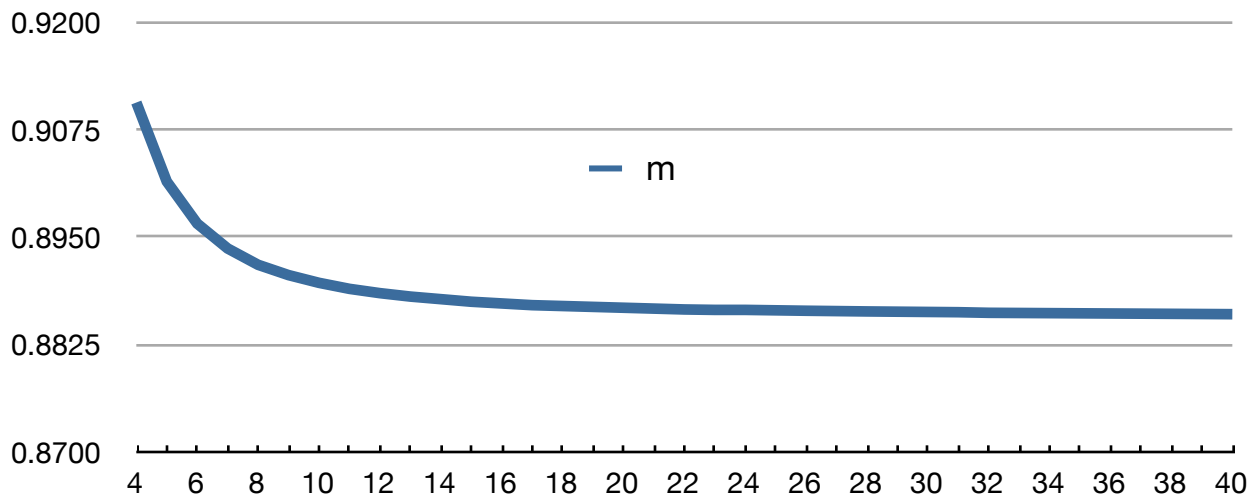


Figure 4 - 3db Bandwidth/Fno vs Average Box Size

Figure 4 shows that, for larger values of N (larger box size, lower output sample frequency), the 3db bandwidth frequency is about 88.6% of the Output Nyquist Frequency. For smaller values of N, this fraction increases to about 91% for N=4.

**Generalized Frequency Response:** Finally, let's consider the frequency response more generally. We have 4 specific frequencies to consider, the output sample frequency ( $1/N$ ) where we know that the system gain is zero. We know that the system gain at zero frequency is 1. We know that the system gain at the Output Nyquist Frequency ( $1/2$  the output sample frequency) is close to 0.637 but varies with  $N$ . And, we know that the system gain is  $1/\sqrt{2}$  at frequencies close to 88% of the Output Nyquist Frequency, but also varies with  $N$ .

What we will do is plot the system gain for various frequencies NORMALIZED to the output sample frequency. This relationship starts from EQN 1:

$$H[f] = \left| \frac{\sin(\pi f N)}{N \sin(\pi f)} \right| \quad \text{EQN 1}$$

but  $f = k/N$

where  $1/N$  represents the output sample rate and  $k$  is the frequency scaler ( $k = 0$  to  $1$ ). This evaluates to

$$H[k] = \left| \frac{\sin(\pi k)}{N \sin(\pi k/N)} \right| \quad \text{EQN 8}$$

Figure 5 shows this in a linear plot (that is, linear frequency axis, linear response axis) for values of  $N=4$ ,  $N=10$ , and  $N=25$  (output sample rates of 25Hz, 10Hz, and 4 Hz, respectively). It should be apparent that the differences between the responses for different values of  $N$  is relatively small (when normalized to the output sample frequency)

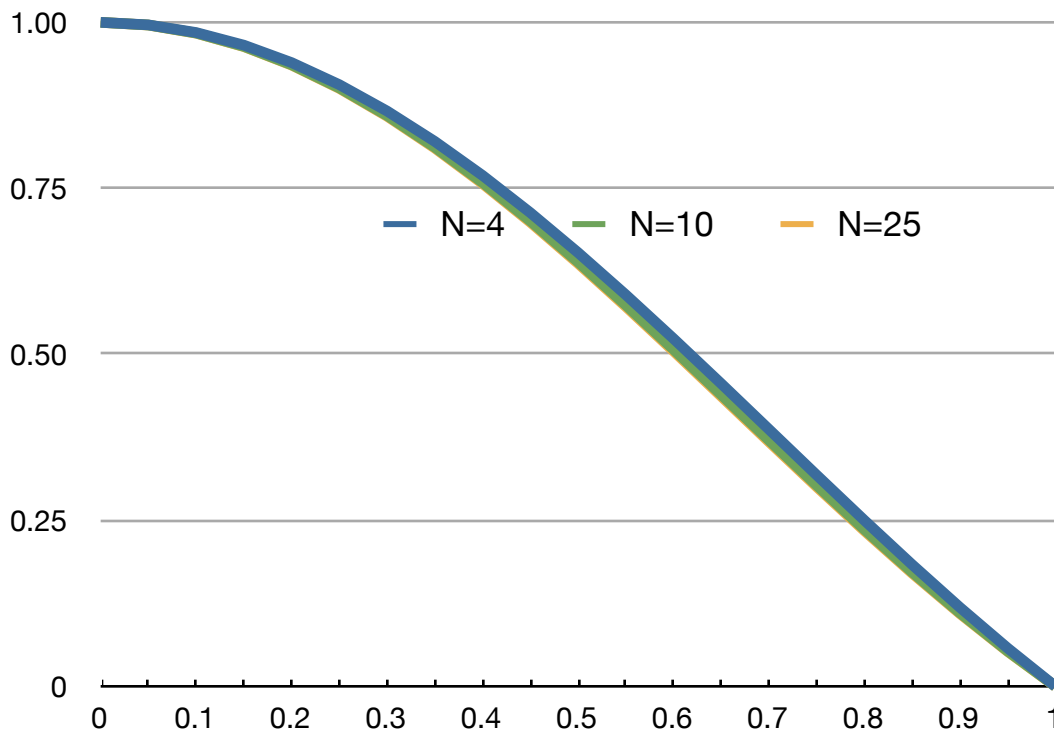


Figure 5 - Normalized Frequency Response for several values of  $N$



## How Does All This Help Set Sample Rate?

The frequency response does not directly tell you how to set the sample rate. But, from the previous discussion, the following basic rules might be stated:

- At a very minimum, you **MUST** set the sample rate to be at least twice the highest expected frequency component. This is the Nyquist Sampling Theorem. If you set the sample rate lower than this, high frequency components may be aliased to a lower frequency and give you false signal indications (and higher than expected noise levels).
- If you are comparing amplitudes of various frequency (spectrum) components, then set the sampling rate to be about 1/0.4 of the highest frequency component where you want the sensor response to be flat within 3db. You **CAN** correct for this high frequency amplitude reduction using EQN 8.

### Summary

It is important to remember that you do not totally lose signals at frequencies above the bandwidth frequency. Such signals are simply reduced to a smaller amplitude, and the reduction is greater as the frequency increases. The output signal does drop to zero for signals that are exactly equal, in frequency, to the output sample frequency.

It is also important to remember that you cannot reproduce any signal very accurately with fewer than 4-8 samples per cycle. The Output Nyquist Frequency is only a limit. It represents the absolute maximum frequency that can be represented at a given sample rate. The Output Nyquist Frequency does **NOT** represent a desirable operating condition!

Finally, since the moving average is linear (scaling all input amplitude values by "k" results in an output value that is also scaled exactly by "k"), input frequencies appear unchanged in the output (for input frequencies lower than the Output Nyquist Frequency). Thus, while the filtering may alter the relative indicated amplitude at various frequencies, the filtering neither adds nor changes nor removes frequencies. Please see the companion Application Note on Aliasing for a discussion of this topic.

### References:

- (1) "The Scientist and Engineer's Guide to Digital Signal Processing", Steven Smith, p 277 (accessed on-line at <http://www.dspguide.com/pdfbook.htm> in Chapter 15, June, 2015)
- (2) *ibid*, Equation 15-2, p 280